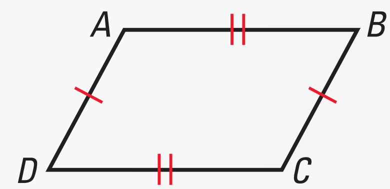
Pre-AP Geometry Date_____ 6.3 Notes

Proving Quadrilaterals are Parallelograms(pp 338-341)

THEOREMS

THEOREM 6.6

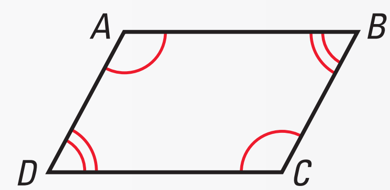
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



***ABCD* is a parallelogram.**

THEOREM 6.7

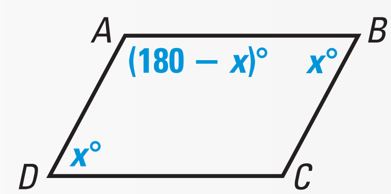
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.



***ABCD* is a parallelogram.**

THEOREM 6.8

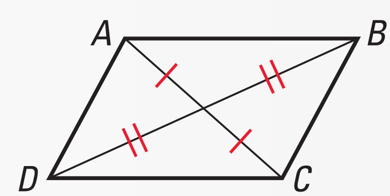
If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram.



***ABCD* is a parallelogram.**

THEOREM 6.9

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

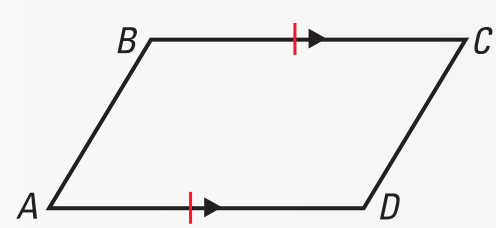


***ABCD* is a parallelogram.**

THEOREM

THEOREM 6.10

If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.



***ABCD* is a parallelogram.**

Proving Quadrilaterals are Parallelograms(pp 338-341)

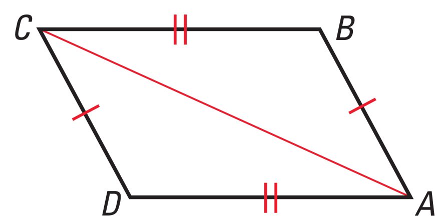
CONCEPT SUMMARY — PROVING QUADRILATERALS ARE PARALLELOGRAMS

- Show that both pairs of opposite sides are parallel.
- Show that both pairs of opposite sides are congruent.
- Show that both pairs of opposite angles are congruent.
- Show that one angle is supplementary to both consecutive angles.
- Show that the diagonals bisect each other.
- Show that one pair of opposite sides are congruent and parallel.

EXAMPLE 1 *Proof of Theorem 6.6*

Proof

Prove Theorem 6.6.



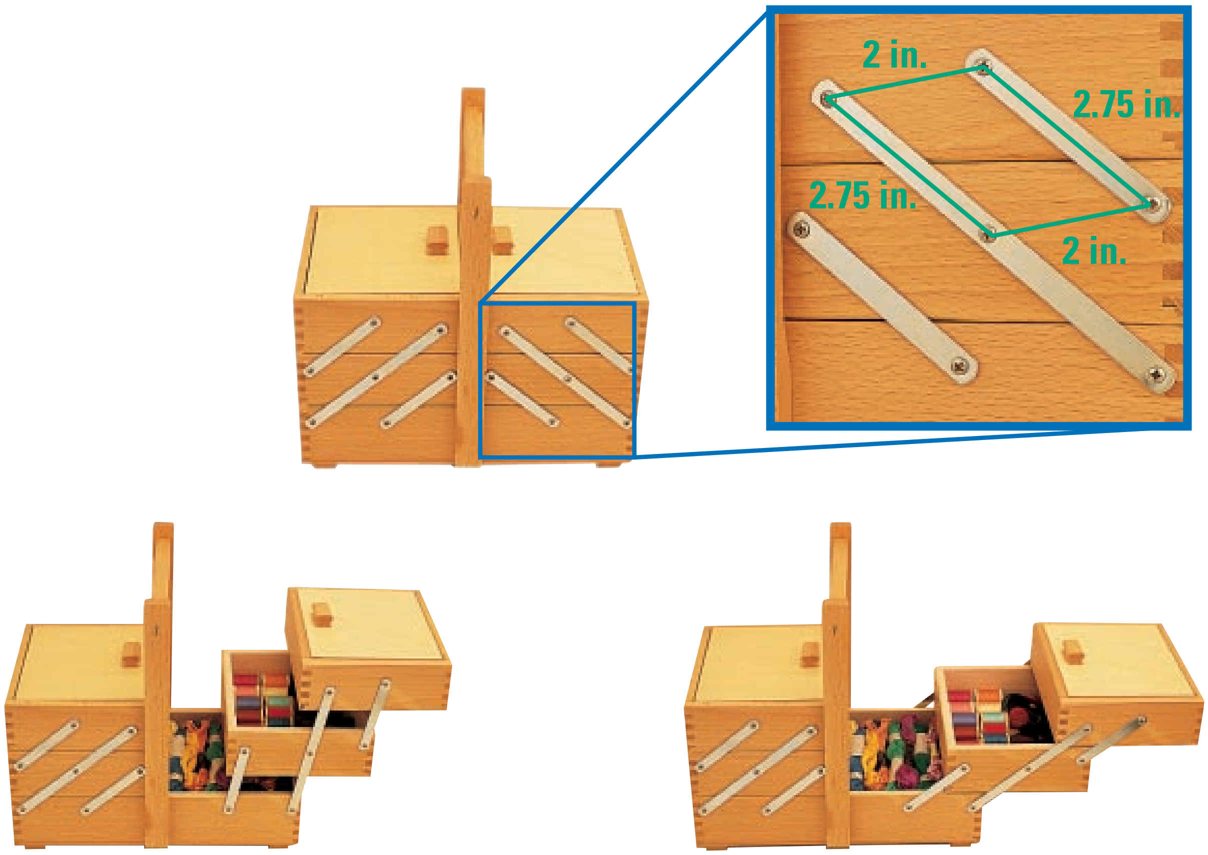
GIVEN ▶ $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$

PROVE ▶ $ABCD$ is a parallelogram.

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$	**1.** Given
2. $\overline{AC} \cong \overline{AC}$	**2.** Reflexive Property of Congruence
3. $\triangle ABC \cong \triangle CDA$	**3.** SSS Congruence Postulate
4. $\angle BAC \cong \angle DCA$, $\angle DAC \cong \angle BCA$	**4.** Corresponding parts of $\cong$ $\triangle$s are $\cong$.
5. $\overline{AB} \parallel \overline{CD}$, $\overline{AD} \parallel \overline{CB}$	**5.** Alternate Interior Angles Converse
6. $ABCD$ is a $\square$.	**6.** Definition of parallelogram

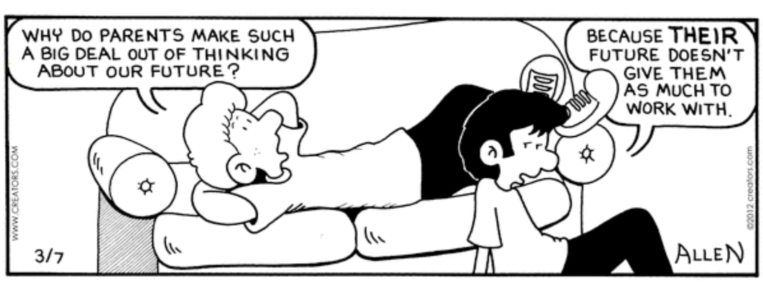
EXAMPLE 2 *Proving Quadrilaterals are Parallelograms*

As the sewing box below is opened, the trays are always parallel to each other. Why?



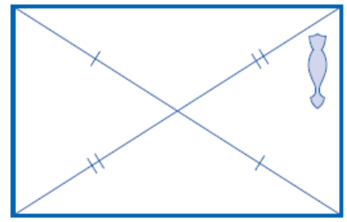
SOLUTION

Each pair of hinges are opposite sides of a quadrilateral. The 2.75 inch sides of the quadrilateral are opposite and congruent. The 2 inch sides are also opposite and congruent. Because opposite sides of the quadrilateral are congruent, it is a parallelogram. By the definition of a parallelogram, opposite sides are parallel, so the trays of the sewing box are always parallel.



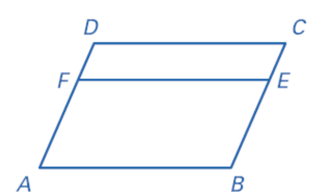
Proving Quadrilaterals are Parallelograms(pp 338-341)

1. A gate is braced as shown. How do you know that the opposite sides of the gate are congruent?



2. Guided Practice.

Given: $ABCD \text{ is a } \square.$
$\overline{FE} \parallel \overline{DC}.$



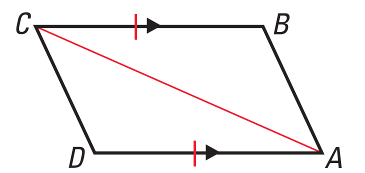
Prove: ABEF is a parallelogram.

Proving Quadrilaterals are Parallelograms(pp 338-341)

EXAMPLE 3 ***Proof of Theorem 6.10***

Proof

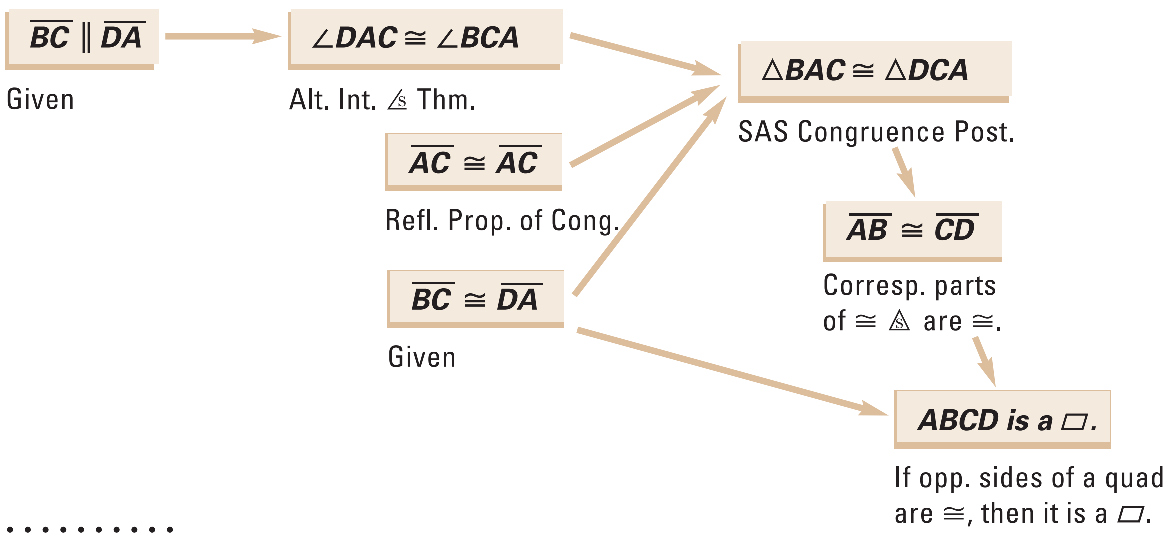
Prove Theorem 6.10.



GIVEN ▶ $\overline{BC} \parallel \overline{DA}$, $\overline{BC} \cong \overline{DA}$

PROVE ▶ $ABCD$ is a parallelogram.

Plan for Proof Show that $\triangle BAC \cong \triangle DCA$, so $\overline{AB} \cong \overline{CD}$. Use Theorem 6.6.



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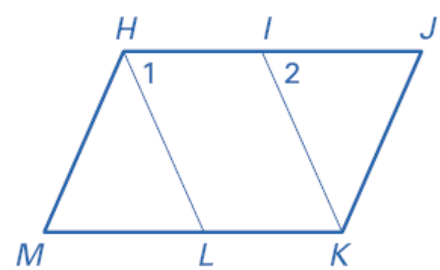
3. Given: $HJKM$ is a ▱.
$\triangle IJK \cong \triangle LMH$

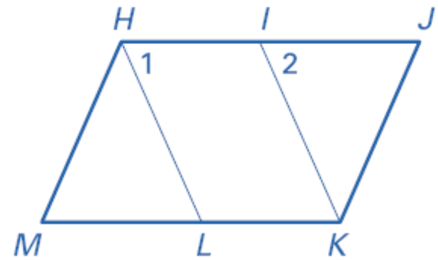
Prove: $HIKL$ is a ▱.



Proving Quadrilaterals are Parallelograms(pp 338-341)

4. Given: $\measuredangle 1 \cong \measuredangle 2$
$\triangle IJK \cong \triangle LMH$

Prove: HIKL is a parallelogram.

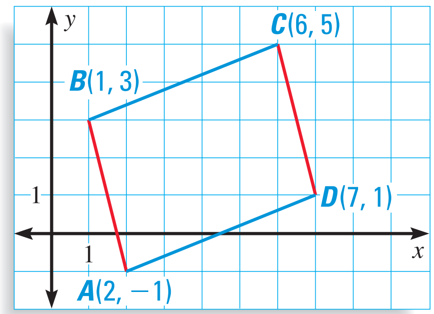


"... so, we agree on salary cuts for everyone except the members of this committee."

Proving Quadrilaterals are Parallelograms(pp 338-341)

EXAMPLE 4 *Using Properties of Parallelograms*

Show that $A(2, -1)$, $B(1, 3)$, $C(6, 5)$, and $D(7, 1)$ are the vertices of a parallelogram.



STUDENT HELP

Study Tip
Because you don't know the measures of the angles of *ABCD*, you can *not* use Theorems 6.7 or 6.8 in Example 4.

SOLUTION

There are many ways to solve this problem.

Method 1 Show that opposite sides have the same slope, so they are parallel.

$$\text{Slope of } \overline{AB} = \frac{3-(-1)}{1-2} = -4$$

$$\text{Slope of } \overline{CD} = \frac{1-5}{7-6} = -4$$

$$\text{Slope of } \overline{BC} = \frac{5-3}{6-1} = \frac{2}{5}$$

$$\text{Slope of } \overline{DA} = \frac{-1-1}{2-7} = \frac{2}{5}$$

$\overline{AB}$ and $\overline{CD}$ have the same slope so they are parallel. Similarly, $\overline{BC} \parallel \overline{DA}$.

▶ Because opposite sides are parallel, *ABCD* is a parallelogram.

Method 2 Show that opposite sides have the same length.

$$AB = \sqrt{(1-2)^2 + [3-(-1)]^2} = \sqrt{17}$$

$$CD = \sqrt{(7-6)^2 + (1-5)^2} = \sqrt{17}$$

$$BC = \sqrt{(6-1)^2 + (5-3)^2} = \sqrt{29}$$

$$DA = \sqrt{(2-7)^2 + (-1-1)^2} = \sqrt{29}$$

▶ $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$. Because both pairs of opposite sides are congruent, *ABCD* is a parallelogram.

Method 3 Show that one pair of opposite sides is congruent and parallel.

Find the slopes and lengths of $\overline{AB}$ and $\overline{CD}$ as shown in Methods 1 and 2.

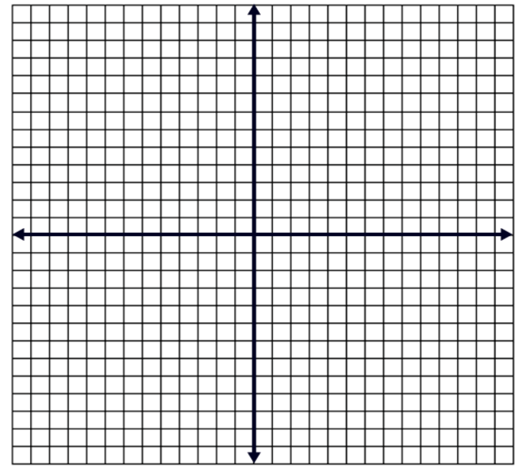
$$\text{Slope of } \overline{AB} = \text{Slope of } \overline{CD} = -4$$

$$AB = CD = \sqrt{17}$$

▶ $\overline{AB}$ and $\overline{CD}$ are congruent and parallel, so *ABCD* is a parallelogram.

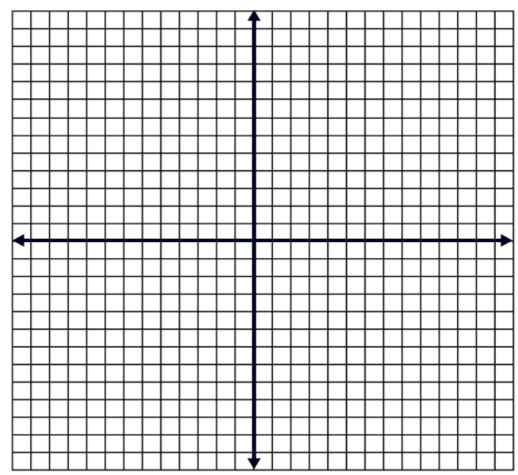
Proving Quadrilaterals are Parallelograms(pp 338-341)

5. Example: Show that A(-1, 2), B(3, 2), C(1, -2), & D(-3, -2) are vertices of a parallelogram.



6. _____ Identify any quadrilateral that is a parallelogram.

- **A.** G(-3, 1), H(4, 1), I(3, 6), & J(-1, 6)
- **B.** P(-2, 2), Q(1, 1), R(4, 4), & S(1, 4)
- **C.** W(3, -1), X(4, 2), Y(1, 5), & Z(0, 2)



7. State the 6 ways to prove that a quadrilateral is a parallelogram.

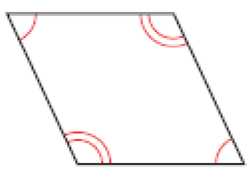
Proving Quadrilaterals are Parallelograms(pp 338-341)

8. In $\square ABCD,$ the ratio of $m\measuredangle A$ to $m\measuredangle B$ is 4:5. What are the measures of all the angles?

9. Is a hexagon with opposite sides parallel called a parallelogram? Explain.

Decide whether you are given enough information to determine that the quadrilateral is a parallelogram. Explain your reasoning.

10.



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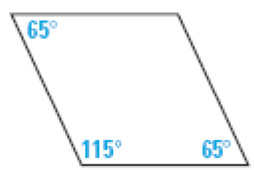

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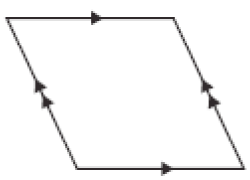

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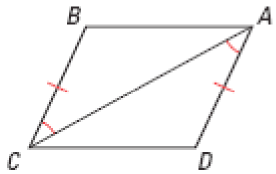
Describe how you would prove that ABCD is a parallelogram.

13.



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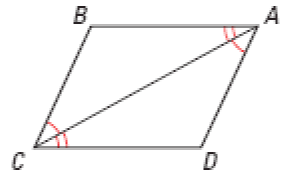

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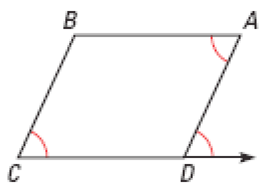

"What do you expect? My edition of the math book doesn't have the answers in it like yours does."